STAT2201, Semester 1 2016

Solution for Quiz #1

For these questions let M be your month of birth, i.e. $M \in \{1, 2, \ldots, 12\}$.

$$M = \_\_\_\_\_$$

Question 1:
Let Z be a standard normal random variable (mean = 0 and variance = 1).

Use the normal distribution table on the back page (or a calculator) to evaluate:

$$\text{Version}_a: \qquad \Pr\big(\,|Z| \,\le \frac{M}{10}\big) = \_\_\_\_\_$$

$$\text{Version}_b: \qquad \Pr\big(\,|Z| \,\ge \frac{M}{10}\big) = \_\_\_\_\_$$

The answer is easily obtained using the normal probability table supplied with the quiz. Note (for e.g.) that,

$$\Pr\big(|Z| \ge x\big) = 2\Pr\big(Z \le -x\big) = 2\Big(1 - \Pr\big(Z \le x\big)\Big).$$

Here are the numerical answers for different M:

	$M=1$	$M=2$	$M=3$	$M=4$	$M=5$	$M=6$	$M=7$	$M=8$	$M=9$	$M=10$	$M=11$	$M=12$
Version_a	0.0796557	0.158519	0.235823	0.310843	0.382925	0.451494	0.516073	0.576289	0.63188	0.682689	0.728668	0.769861
Version_b	0.920344	0.841481	0.764177	0.689157	0.689157	0.548506	0.483927	0.423711	0.36812	0.317311	0.317311	0.317311

The level of accuracy above is obtained using software. Using the supplied table would yield lower accuracy, but that is acceptable.

Note: Observe the trend in M.

Observe that the sum of version a and version b answers is 1.

Question 2: Assume a sample with 4 observations: $\{M, M^2, 3M, \frac{M}{2}\}$ (this is what was given in version a - version b is slightly different).

(a) Sample mean = _____

$$\overline{Y} = \frac{1}{4}\sum_{i=1}^{4} Y_i = \frac{1}{4}M\big(1 + M + 3 + \frac{1}{2}\big) = \frac{1}{4}M(4.5 + M).$$

Only a numerical answer was expected - to see it, just plug in M in the above.

(b) Sample variance = _____

You need to remember the formula (taking the sample size as 4):

$$S^2 = \frac{1}{4-1}\sum_{i=1}^{4}\big(\overline{Y} - Y_i\big)^2 = \frac{\big(\sum_{i=1}^{4} Y_i^2\big) - 4\overline{Y}^2}{3}.$$

Plugging in the values we obtain:

$M=1$	$M=2$	$M=3$	$M=4$	$M=5$	$M=6$	$M=7$	$M=8$	$M=9$	$M=10$	$M=11$	$M=12$
1.22917	4.91667	15.5625	43.6667	105.729	105.729	427.729	750.667	1233.56	1922.92	2871.23	4137

(c) Draw a rough sketch of the Empirical CDF for this sample:

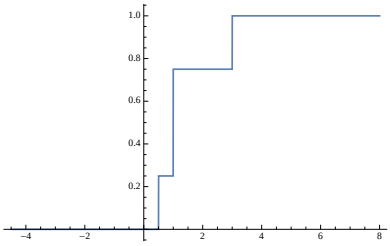
The ECDF is a function of x:

$$F(x) = \frac{1}{n}A(x),$$

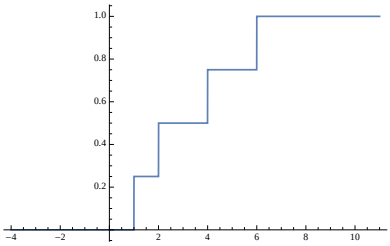
where $A(x)$ is the number of observations less than or equal to x and n is the number of observations ($n = 4$ in this case). Thus for x less than the minimal observation the ECDF is 0 and for x equal or greater than the largest observation, the ECDF is 1. In between it jumps, each time at a step of size $1/n$ at observation points.

Rough sketches of the ECDF (not taking into account the discontinuity at observation points) are below:

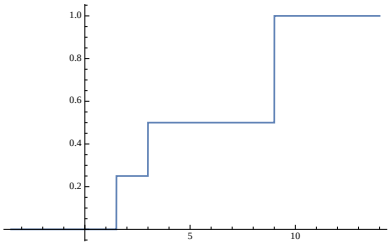
$M = 1$:



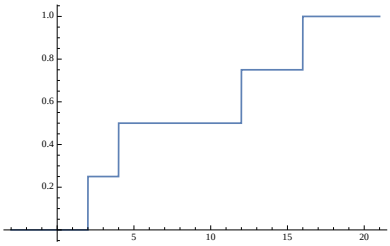
$M = 2$:



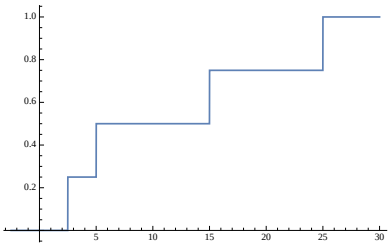
$M = 3$:



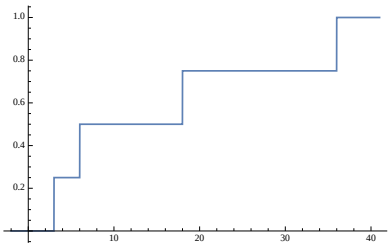
$M = 4$:



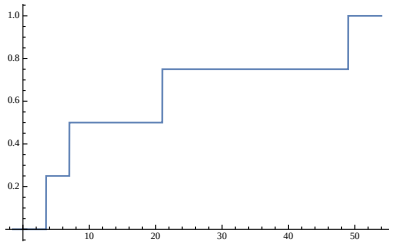
$M = 5$:



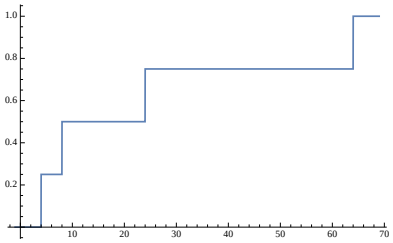
$M = 6$:



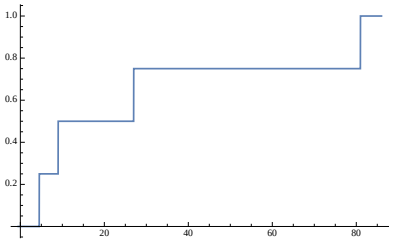
$M = 7$:



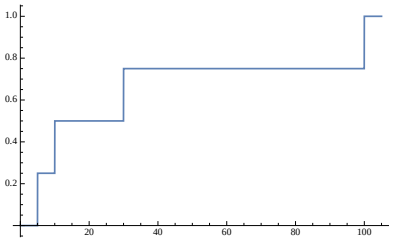
$M = 8$:



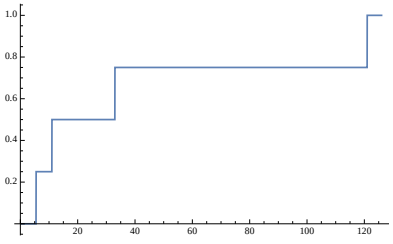
$M = 9$:



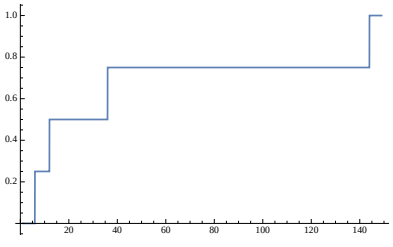
$M = 10$:



$M = 11$:



$M = 12$:



NORMAL CUMULATIVE DISTRIBUTION FUNCTION: $\Pr(Z \leq x)$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000