Class Example 1. Hello World in Julia

The purpose of this exercise is to gain familiarity with the Julia interface.

(a) Log into JuliaBox.

(b) Solve 1+1 in Julia.

(c) Now let's explore this simple thing: What numbers x satisfy the property of having the sum of x with itself be the same as the product of x and itself? We'll solve this simple problem in a few ways:

(i) By solving the quadratic equation, $x+x = x*x$ analytically (rearrange it to $x(x-2) = 0$), we obtain the solutions $x = 0$ and $x = 2$.

(ii) Use the quadratic formula to find the solution in Julia, even though it isn't needed:

$$x^2 - 2x = 0 \qquad \Rightarrow \qquad x = \frac{2 \pm \sqrt{(-2)^2}}{2}.$$

Key this into Julia to solve, use the function sqrt().

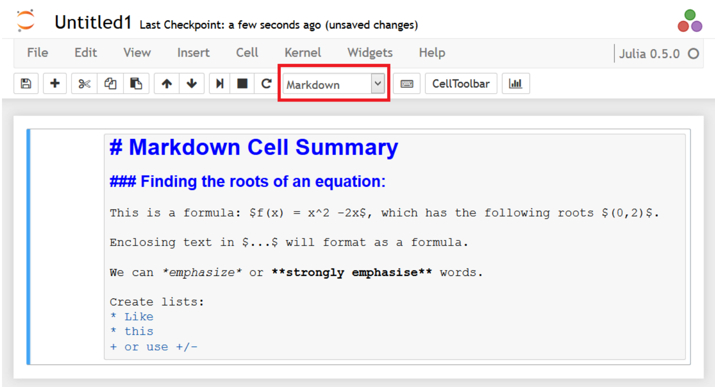
```
[(2 + sqrt(2^2))/2, (2 - sqrt(2^2))/2]
```

(iii) We can solve this numerically in Julia as follows:

```
using Roots
f(x) = x^2-2*x
fzero(f,-1,1) , fzero(f,1,4)
```

(d) Describe what you have just done in an IJulia notebook. Transform your notebook to a PDF file, and save the file.

The IJulia notebook is extremely flexible, as individual cells can be either code cells which can contain executable code, or text cells (using markdown). Markdown allows for fast formatting of text, and also provides the ability to include scientific formulas using "LaTeX" syntax. An example of a markdown cell is shown below.



(e) Take a photo of handwritten material with your cellular device or similar. Then upload the photo to your computer. Then set a notebook cell as markdown. Now drag the photo file onto the cell. Finally, hit Shift + Enter to evaluate the cell and see the photo in the notebook.

Now choose File → Download → HTML Embedded (.html). This will open a new browser tab with your formatted output. Save this tab as a PDF (e.g. Print to PDF).

Class Example 2. The Sum of Two Dice

You are rolling two independent, fair, six sided dice. Answer each of the following
(i) Analytically, (ii) Using probability calculations in Julia, (iii) Using Monte-Carlo in Julia.

(a) What is the probability of the sum of the outcomes being even?

(b) What is the probability of the *product* of the outcomes being even?

(c) What is the set of the least likely outcomes for the sum?

Solution:

(a)

(i) The table below enumerates all possible outcomes and contains the sum associated with each outcome. The even sums are shaded. There are 36 outcomes in total of which 18 are even. Since all outcomes have equal probability,

$$\text{probability of even sum} = \frac{\text{number of even sum outcomes}}{\text{total number of outcomes}} = \frac{18}{36} = 0.5.$$

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(ii) We can calculate this numerically in Julia as follows:

```
sum([1 - (i+j)%2 for i in 1:6, j in 1:6])/36
```

(iii) We can use a Monte Carlo simulation to approximate this in Julia as follows:

```
x = [1,2,3,4,5,6]
sum([1 - (rand(x)+rand(x))%2 for _ in 1:10^4]) / 10^4
```

(b)

(i) Using similar counting reasoning as in (a)(i), observe the table below to see that the probability of the product of the outcomes being even is 0.75.

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

(ii) We can calculate this numerically in Julia as follows:

```
sum([1 - (i*j)%2 for i in 1:6, j in 1:6])/36
```

(ii) We can use a Monte Carlo simulation to approximate this in Julia as follows:

```
x = [1,2,3,4,5,6]
sum([1 - (rand(x)*rand(x))%2 for _ in 1:10^4])/10^4
```

(c) We can consider outcomes of the experiment as being the sum of the dice. Then the outcomes are $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. In this case the probability of each outcome is no longer equal (as is for a roll of a single die). By looking at the table of (a)(i) we see that the following probabilities hold:

$$\begin{aligned}
P(\text{sum is } 2) &= 1/36, \\
P(\text{sum is } 3) &= 2/36, \\
P(\text{sum is } 4) &= 3/36, \\
P(\text{sum is } 5) &= 4/36, \\
P(\text{sum is } 6) &= 5/36, \\
P(\text{sum is } 7) &= 6/36, \\
P(\text{sum is } 8) &= 5/36, \\
P(\text{sum is } 9) &= 4/36, \\
P(\text{sum is } 10) &= 3/36, \\
P(\text{sum is } 11) &= 2/36, \\
P(\text{sum is } 12) &= 1/36.
\end{aligned}$$

Observe that the sum of all probabilities is 1. We thus see that obtaining either 2 or 12 are the least likely events.

Class Example 3. Cellular Phones for All

An Australian not-for-profit organization is collecting cellular devices for shipment to schools in a third world country (the devices will be used for education). In an initial sample, 1000 working devices are collected and are tested for both,

$$A \equiv \text{Surface Flaws}, \qquad \text{and} \qquad B \equiv \text{Defective Camera}.$$

The results are as follows:

	$\boldsymbol{A}$	$\boldsymbol{\overline{A}}$
$\boldsymbol{B}$	109	364
$\boldsymbol{\overline{B}}$	468	59

Using the above sample, give an estimate for the following:

(a) $P(A \cap B)$ (also denoted, $P(A, B)$).

(b) $P(A \mid B)$.

(c) $P(A)$ and $P(B)$.

(d) The company plans now to collect $100,000$ devices. How many devices are expected to be collected without surface flaws and a working camera?

(e) Given that a device is collected with no surface flaws, what is the chance that it has a defective camera?

Solution:

(a) Probability of Surface flaws and Defective camera: $109/1000 = 0.109$.

(b) Probability of surface flaws, given a defective camera: $109/(109 + 364) = 0.2304$. Note that in the course, we give answers of probabilities with a precision of four decimal places.

(c) Probability of Surface flaws: $(109 + 468)/1000 = 0.577$.
Probability of Defective Camera: $(109 + 364)/1000 = 0.473$.

(d) $100,000 \times P(\overline{A} \cap \overline{B}) = 100,000 \times 59/1000 = 5,900$.

(e) $P(B \mid \overline{A}) = 364/(364 + 59) = 0.8605$.

Question 1. A Digital Scale

A digital scale that provides weights to the nearest gram is used.

(a) What is the sample space for this experiment?

Let A denote the event that a weight exceeds 12 grams, let B denote the event that a weight is less than or equal to 14 grams, and let C denote the event that a weight is greater than or equal to 9 grams and less than 14 grams. Describe the events:

(b) $A \cup B$

(c) $A \cap B$

(d) $\overline{A}$

(e) $A \cup \big(B \cap C\big)$

(f) $\overline{(A \cup C)}$

(g) $A \cap B \cap C$

(h) $\overline{B} \cap C$

Question 2. Transmitting Bits

Four bits are transmitted over a digital communications channel. Each bit is either distorted or received without distortion. Let A_i denote the event that the i^{th} bit is distorted, $i = 1, \ldots, 4$.

(a) Describe the sample space for this experiment.

(b) Are the $A_i's$ mutually exclusive?

Describe the outcomes in each of the following events:

(c) A_1

(d) $\overline{A_1}$

(e) $A_1 \cap A_2 \cap A_3 \cap A_4$

(f) $(A_1 \cap A_2) \cup (A_3 \cap A_4)$

Question 3. Basic Simulation With Julia

The sample space of a random experiment is $\{a, b, c, d, e\}$ with probabilities $0.1, 0.1, 0.2, 0.4, 0.2$ respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

(a) $P(A)$

(b) $P(B)$

(c) $P(\overline{A})$

(d) $P(A \cup B)$

(e) $P(A \cap B)$

Below are two alternative Julia code blocks. In each block we generate samples with values "1", "2", "3" or "4" based on the weights, $0.1, 0.6, 0.2$ and 0.1 respectively. Notice that the code blocks differ.

```
#code block (1)

using StatsBase
w = weights([0.1, 0.6, 0.2, 0.1])

prop = sum([sample(w) == 1 || sample(w) == 3 for _ in 1:10^6])/10^6
```

```
#code block (2)

using StatsBase
w = weights([0.1, 0.6, 0.2, 0.1])

function eventHolds(wInstance)
    wInstance == 1 || wInstance == 3
end

prop = sum([eventHolds(sample(w)) for _ in 1:10^6])/10^6
```

(f) Run code block (1) and code block (2). Explain the difference in the results. Describe the value of "prop". Can you explain why the output of "prop" is what it is? Hint: Calculating the expected value of "prop" (by hand) for code block (2) is much more straight forward than code block (1).

(f) Modify either code block (1) or code block (2) (choose the correct one), to simulate the experiment of the question (with $\Omega = \{a, b, c, d, e\}$) using 10^6 replications. Based on the simulation runs, present your estimates for the probabilities in (a)–(e) and compare then to your exact answers for (a)–(e).

Question 4. NiCD Battery

In an NiCd battery, a fully charged cell is composed of Nickelic Hydroxide. Nickel is an element that has multiple oxidation states and that is usually found in the following states:

Nickel Charge	**Proportions Found**
0	0.12
+2	0.40
+3	0.30
+4	0.18

For both of these items, formulate the events as sets as you present your answer.

(a) What is the probability that a cell has at least two of the positive nickel-charged options?

(b) What is the probability that a cell is not composed of a positive nickel charge greater than +2?

Question 5. Hacking the NSA

A computer system uses passwords that contain exactly eight characters, and each character is one of 26 lower-case letters (a-z) or 26 upper-case letters (A-Z) or 10 integers (0-9). Let Ω denote the set of all possible passwords, and let A and B denote the events that consist of passwords with only letters or only integers, respectively. Suppose that all passwords Ω are equally likely. Determine the probability of each of the following:

(a) A

(b) B

(c) A password contains at least 1 integer.

(d) A password contains exactly 2 integers.

The following Julia code generates 100 random passwords and counts how many of them contain 1 or less lower case letters.

```
passLength = 8
numToCheck = 1
possibleChars = ['a':'z';'A':'Z';'0':'9']

#Define a function that counts how characters are lower case
numLowerCaseChars(str) = sum([islower(char) for char in str])

n = 100
passwords = [String(rand(possibleChars,passLength)) for _ in 1:n]
proportion = sum([numLowerCaseChars(p) <= numToCheck for p in passwords])/n
```

(e) In your view, are 100 passwords sufficient for obtaining a sensible estimate for the event of having 1 or less lower case characters? Modify the code to obtain a more accurate estimate.

(f) Modify the code to obtain estimates for the probabilities of the events in (a)–(d). Compare with your theoretical results. You may want to use the "isnumber()" function. e.g. isnumber('7').

Question 6. Cast Aluminium

Samples of a cast aluminium part are classified on the basis of surface finish (in microinches) and length measurements. The results of 200 parts are summarized as follows:

		Length	
		Excellent	Good
Surface	Excellent	140	5
Finish	Good	30	25

Let A denote the event that a sample has excellent surface finish, and let B denote the event that a sample has excellent length. Determine:

(a) $P(A)$

(b) $P(B)$

(c) $P(A \mid B)$

(d) $P(B \mid A)$

(e) If the selected part has excellent surface finish, what is the probability that the length is excellent?

(f) If the selected part has good length, what is the probability that the surface finish is excellent?

Question 7. Cotton Fabric and Nylon Fabric

Suppose 2% of cotton fabric rolls and 3% of nylon fabric rolls contain flaws. Of the rolls used by a manufacturing company, 20% are cotton and 80% nylon. What is the probability that a randomly selected roll used by the company contain flaws?

Question 8. Computer Keyboard Failure

Computer keyboard failures are due to faulty electrical connects (10%) or mechanical defects (90%). Mechanical defects are related to loose keys (22%) or improper assembly (78%). Electrical connect defects are caused by defective wires (35%), improper connections (13%), or poorly welded wires (52%).

(a) Find the probability that a failure is due to loose keys.

(b) Find the probability that a failure is due to improperly connected or poorly welded wires.