Question 1. Magic of the CLT

Let $X_i \sim Bin(5, 0.1)$ for $i = 1, 2, \ldots$ independently distributed. Let $S_n = \sum_{i=1}^{n} X_i$.

(a) Use R to plot S_1, S_{10}, S_{30} and comment on your plots.

(b) What is (approximately) the distribution of S_{100}?

Question 2. Testing Errors

A textile fibre manufacturer is investigating a new drappery yearn, which the company claims has a thread elongation of 12kg with a known standard deviation of 0.5kg. The company wishes to test the claim of the mean, believing the standard deviation to be true.
The hypothesis H_0 is $\mu = 12$ and the alternative is chosen to be $H_1 : \mu < 12$ with a rejection region of $\mathcal{R} = \{\bar{x} < 11.5\}$. Suppose 4 samples of specimens are taken for the testing.

(a) What is the type I error probability α?

(b) Find the type II error probability β, if the true mean is in fact 11.25kg.

Question 3. Confidence

Let X_i be identically and independently distributed with unknown mean μ and known variance $\sigma = 0.2$. Suppose 16 samples are taken and 3.3 is its sample mean. Find the following probabilities:

(a) $P\left(|\bar{x} - \mu| \leq 1.5\frac{0.2}{\sqrt{16}}\right)$

(b) Suppose now N samples are taken and $\bar{x}$ is its sample mean. Determine the probability that the distance between $\bar{x}$ and μ is at most $1.8\frac{\sigma}{\sqrt{N}}$.

Question 4. P-Values

For the hypothesis test $H_0 : \mu = 7$ against $H_1 : \mu \neq 7$ and variance known, calculate the P-values for the test statistic $z = 2.05$.

Question 5. Hypothesis Testing

The life in hours of a battery is known to be approximately normally distributed, with standard deviation $\sigma = 1.25$ hours. A random sample of 10 batteries has a mean life of $\bar{x} = 40.5$ hours.

(a) Is there evidence to support the claim that battery life exceeds 39.5 hours? Use $\alpha = 0.05$.

(b) What sample size would be required to ensure that β (Type II error) does not exceed 0.12 if the true mean life is 40.8 hours?

Question 6. Explore the Student's t-Distribution

Let us use R to become familiar with the Student's t-distribution. For that you may have to use some of the following (R-inbuilt) functions:

```
rt(n, df)     # random generator for distribution
?rt           # information on rt and its inputs
```

Take 10^4 samples of a Student's t-distribution with 3 degrees of freedom. Use a *qnorm* plot to test the relation to a normal distribution. Comment on your result.

Question 7. t-Test

A report based on a study conducted in 2003, reported on the body temperature (in Fahrenheit) of 25 females. The values are given in *5-7.csv*.
Test the hypothesis $H_0 : \mu = 98.6$ versus $H_1 : \mu \neq 98.6$, using $\alpha = 0.05$.

Question 8. Applied t-Test

The sodium content of fifteen 300-gram boxes of organic cornflakes was determined. The data (in milligrams) is contained in *5-8.csv*. Can you support a claim that mean sodium content of this brand of cornflakes is higher than 115 milligrams? (use $\alpha = 0.05$, state your hypothesis clearly and make a conclusion.)