Question 1 – Web Page Counter

Consider an application that counts the number of web pages visited on a server on a specific day.

Let A denote the event that at least 20 web pages were visited. Let B denote the event that the number of visited web pages is less than or equal to 37, and let C denote the event that at least 45 web pages were visited that day.

(a) What is the sample space for this experiment?

(b) Are the events A, B, C mutually exclusive?

Describe the events:

(c) $A \cap B$

(d) $\overline{A}$

(e) $A \cup C$

(f) $\overline{B} \cap C$

(g) $A \cup \left(\overline{B} \cap C\right)$

(h) $\overline{(A \cup C)}$

(i) $A \cap B \cap \overline{C}$

Question 2 – Transmitting Bits

Four bits are transmitted over a digital communications channel. Each bit is either distorted or received without distortion. Let A_i denote the event that the ith bit is distorted, $i = 1, \ldots, 4$.

(a) Describe the sample space for this experiment.

(b) Are the A_i's mutually exclusive?

Describe the outcomes in each of the following events:

(c) A_3

(d) $\overline{A_2}$

(e) $A_1 \cap A_2 \cap \overline{A_3} \cap A_4$

(f) $(A_1 \cup A_2) \cap (A_3 \cap A_4)$

(g) $(\overline{A_1 \cap A_2}) \cup (\overline{A_3} \cap A_4)$

Question 3 – Basic Simulation with R

The sample space of a random experiment is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.05, 0.4, 0.2, 0.25 respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

(a) $P(A)$

(b) $P(B)$

(c) $P(\overline{A})$

(d) $P(A \cup B)$

(e) $P(A \cap B)$

The following R Code obtains a sample with values "1","2","3","4" chosen with weights of 0.1, 0.6,0.2,0.1 respectively:

```
> val <- 1:4
> weig <- c(0.1, 0.6,0.2,0.1)
> Wsam <- sample(val,1,prob = weig)
> print(Wsam)
```

(f) Run the following R block:

```
> val <- 1:4
> weig <- c(0.1, 0.6,0.2,0.1)
> prop <- mean(replicate(10^6,any(c(1,3)==sample(val,1,prob = weig))))
> print(prop)
```

Explain what this R block is doing, i.e., what does the value of "prop" represent?

(g) Modify the code in f) to simulate the experiment of the question (with $\Omega = \{a, b, c, d\}$) using 10^6 replications. Based on the simulation runs, present your estimates for the probabilities in (a)-(e) and compare them to your exact asnwers for (a)-(e).

Question 4 – Car Features

BMW offers their cars as base models where clients can add different sets of features (e.g. screens on backseats, integrated GPS). Suppose, the following table lists the proportion of BMW cars bought in Australia in 2018 dependent on their features.

Features	**Proportions bought in 2018**
basic model	0.32
basic model with one feature	0.30
basic model with two features	0.22
basic model with three features	0.16

When answering the following questions, also formulate the events considered as sets.

(a) What is the probability that a BMW bought in 2018 has at least two features?

(b) What is the probability that a BMW does not contain more than one feature?

Question 5 – Hacking the NSA

A computer system uses passwords that contain exactly eight characters, and each character is one of 26 lower-case letters (a-z) or 26 upper-case letters (A-Z) or 10 integers (0-9). Let Ω denote the set of all possible passwords. Suppose that all passwords Ω are equally likely. Determine the probability of each of the following:

(a) The password contains only letters.

(b) The password contains at least one integer

(c) A password contains exactly two integers and one lower-case letter.

The following R code generates 100 random passwords and counts how many of them contain 1 or less lower case letters.

```
> poss_values <- c(0:9,letters[1:26],LETTERS[1:26])
>
> # function to count lower case values
> count_prop <- 0
>
> # 100 samples
> it <- 100
>
> for (i in 1:it) {

+          Sample_password <- sample(poss_values,8,replace=TRUE)

+          count_lower <- sum(Sample_password %in% letters)

+          count_prop <- count_prop + (count_lower <= 1)
+ }
>
> print(count_prop/it)
```

(d) In your view, are 100 passwords sufficient for obtaining a sensible estimate for the event of having 1 or less lower case characters? Modify the code to obtain a more accurate estimate.

(e) Modify the code to obtain estimates for the probabilities of the events in (a)-(c). Compare it with your theoretical results.

Question 6 – Air Conditioning Systems

A maintenance firm has gathered the following information regarding the failure mechanisms for air conditioning systems:

		Gas Leaks	
		Yes	No
Electric Leaks	Yes	55	17
	No	32	3

Let A denote the event that an air condition has a gas leak, and let B denote the event that an air condition has an electric leak. Determine:

(a) $P(A)$

(b) $P(\overline{B})$

(c) $P(A \mid B)$

(d) $P(B \cap \overline{A})$

(e) If the selected air condition doesn't have a gas leak, what is the probability that it has an electric leak?

(f) If the selected air condition has a gas leak, what is the probability that it does not have an electric leak?

Question 7 – Flaws in Research Journals

Suppose 1.8% of articles in physics journals and 3% of articles in engineering journals contain flaws. Of the articles read by a PhD student in Engineering, 38% are from physics journals and 62% are engineering articles. What is the probability that a randomly selected article by an engineering PhD student contains flaws?

Question 8 – Computer Keyboard Failure

Computer keyboard failures are due to faulty electrical connects (13%) or mechanical defects (87%). Mechanical defects are related to loose keys (22%) or improper assembly (78%), Electrical connect defects are caused by defective wires (27%), improper connections (26%), or poorly welded wires (47%).

(a) Find the probability that a failure is due to loose keys.

(b) Find the probability that a failure is due to improperly connected or poorly welded wires