Question 1 – Continuous Distribution

Given the cdf F for a continuous random variable X:

$$F(x) = \begin{cases} 0 & x < 0, \\ 1 - e^{-0.01x} & x \ge 0, \end{cases}$$

- (a) Determine P(X > 50).
- (b) Determine $P(X \leq 100)$.
- (c) Determine $P(-10 \leq X \leq 10)$.
- (d) Derive the corresponding probability density function f(x) for X.
- (e) Find the mean.
- (f) Find the standard deviation.

Question 2 – Discrete Distribution

Consider a function p such that

x	10	11	13	16	20
p(x)	0.08	0.15	0.30	0.20	0.27

(a) Verify that p is a probability mass function (pmf).

Calculate the following:

(b)
$$P(X \le 13)$$
,

(c)
$$P(X > 13)$$
,

(d)
$$P(X = 11),$$

- (e) $P(X < 20 \text{ or } X \ge 13)$,
- (f) the mean,
- (g) the variance.

Question 3 – Computer Fans

Suppose that the time to failure (in hours) of fans in a personal computer can be modelled by an exponential distribution with $\lambda = 0.0003$.

- (a) What proportion of fans will last at least 12,000 hours?
- (b) What proportion of fans will last at most 8,000 hours?
- (c) What is the variance of the time until a computer fan fails?
- (d) Use Monte Carlo simulation to predict the following: Assume a computer now has three independent fans and the failure of the computer occurs once all three fans are broken. What is the mean life of the computer? (hint: rexp(n,lambda) generates n random exponentially distributed numbers)

Question 4 – **Guessing on Multiple Choice Exams**

A multiple-choice test contains 30 questions, each with 5 answers. Only one answer is correct. Assume that a student just guesses on each question.

- (a) What is the probability that the student answers more than 15 questions correctly?
- (b) What is the probability that the student answers fewer than 10 questions correctly?

The following code generates the vector "Binvec" = pmf of a Binomial distribution with parameters n = 10 and p = 0.6. It then sums up the vector, illustrating that the sum of all of the probabilities is 1.

```
> Binvec <- dbinom(0:10,10,0.6)
> sum(Binvec) # => 1
```

(c) Modify the code above, to validate your answers in (a) and (b).

Question 5 – Aerospace Inspections

The thickness of a flange on an aircraft component is Uniformly distributed between 0.7 and 1.2 millimetres. Determine the following:

- (a) Cumulative distribution function of flange thickness.
- (b) Proportion of flanges that exceeds 0.93 millimetres.
- (c) Probability that the thickness exceeds 67% of the flanges.
- (d) Mean and standard deviation of flange thickness.
- (e) Assume now that you are sampling 12 independent flanges. What is the variance of the number of flanges with a thickness less than 0.76 millimetres?

Question 6 – The Prototype Shoe

The weight of a sophisticated running shoe is normally distributed with a mean of 12 ounces and a standard deviation of 0.7 ounce.

- (a) What is the probability that a shoe weighs more than 14 ounces?
- (b) What must the standard deviation of weights be in order for the company to state that 99% of its shoes weighs less than 14 ounces?
- (c) If the standard deviation remains at 0.7 ounce, what must the mean weight be for the company to state that 99% of its shoes weigh less than 14 ounces?

Question 7 – Water Pollution

Each sample of water has a 10% chance of containing a particular organic pollutant. Assume you take a sample every hour and that the samples are independent with regard to the presence of the pollutant.

- (a) Find the probability that in the next 15 samples, exactly 2 contain the pollutant.
- (b) Find the probability that in the next 18 samples, more than 4 contain the pollutant.
- (c) What is the mean number of samples with pollutant during 123 samples?

${ Question \ 8-Soda \ Machine }$

The fill volume of a soda can in a soda-machine is normally distributed with a mean of 0.33 litres and a standard deviation of 0.01 litres.

- (a) What is the probability a fill volume is less than 0.325 litres?
- (b) If all cans less than 0.315 or greater than 0.335 litres are scrapped, what proportion of cans is scrapped?
- (c) What is the critical litre amount where 90% of cans fall below?