THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Analysis of Engineering and Scientific Data

Semester 1 – 2019

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Review of Chapter 2

- **Sample space:** Ω.

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- **Operation Laws and DeMorgan Laws**.
  a) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
  b) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
  c) $\overline{A \cup B} = \overline{A} \cap \overline{B}$
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- $0 \leq P(A) \leq 1$,
- $P(\Omega) = 1$,
- if $\{A_i\}$ are disjoint, then $P\left(\bigcup_i A_i\right) = \sum_i P(A_i)$.

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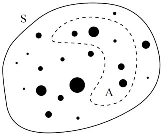

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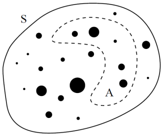
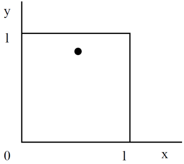


  b) In the continuous case it will (generally) result in a solution of finding an area (an integration) problem:



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- **Independence**:

$$A, B \text{ independent iff } P(A \mid B) = P(A).$$

$$\leftrightarrow P(B \mid A) = P(B).$$

$$\leftrightarrow P(A \cap B) = P(A)\,P(B).$$

Example - independence

- Getting a 7 on the assignments (event A) if one visits the tutorials (event B)? dependent
- Being enrolled in either Calculus (event A) or Statistics (event B) in one semester. dependent
- Drawing a red ball as second ball (event A) if the first ball was blue (event B), if the balls are not returned to the bucket of coloured balls. dependent
- Getting a 2 (event A) or a 4 (event B) when rolling a dice. dependent
- Drawing a red ball as second ball (event A) if the first ball was blue (event B), if the balls are returned to the bucket of coloured balls. independent

Independence vs. Mutually Exclusive

If A, B are mutually exclusive $\longrightarrow$ A, B are independent?
NO!
For example: Rolling a dice once.
A = event to get a '2', B = event to get a '4'.
A,B are mutually exclusive but dependent since
$P(A|B) = 0$ [if you got a '4' you can not get a '2'], but $P(A) = \frac{1}{6}$, so

$$P(A|B) \neq P(A) \quad \longrightarrow A, B \text{ are dependent}$$

If A, B are mutually exclusive $\longleftarrow$ A, B are independent?
NO!
For example: Card game with 52 cards.
A = event to get a spade, B = event to get a ace.
A,B are independent since

$$\frac{1}{52} = P(A \cap B) = \frac{4}{52}\frac{1}{4} = P(B)P(A)$$

but they are not mutually exclusive because there is a card ace in spade.

The Birthday problem

What is the probability that among N students at least 2 students share the same birthday?

$P(22 \text{ students share BD}) = 1 - P(\text{no students share their BD}) =$
$1 - \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \dots \cdot \frac{365-N+1}{365} = 1 - \frac{365!}{(365-N)!} \cdot \frac{1}{365^N}$

Recall: $365! = 365 \cdot 354 \cdot 363 \cdots 2 \cdot 1$

Optional: See RMarkdown document in Lecture 2 folder (blackboard and course webpage) to see this example coded example in R.

Chapter 3

1. Random Variables
2. Probability Mass/Density Function
3. Cumulative Distribution Function
4. Expectation and Variance
5. Special Discrete Distributions (Bernoulli, Binomial,Geometric)
6. Special Continuous Distributions (Uniform, Exponential, Normal)

Random Variables

Definition

A function $X(\omega)$ assigning to every outcome ω in the sample space Ω a real number is called a **random variable**.

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Example:

- Tossing a coin: $\Omega = \{H, T\}$; X could be $X(H) = 1$ (e.g. "Head is success") and $X(T) = 0$ (e.g. "Tail is failure").
- N transistors, a_i = event that i^{th} transistor works; X could assign a 1 if the transistor works, else 0, i.e., $X(a_i) = 1, \quad X(\bar{a}_i) = 0$.
- N transistors, a_i = event that i^{th} transistor works. Take 2 transistors, then the possible outcomes are $\Omega = \{(a_1, a_2), (\bar{a}_1, a_2), (a_1, \bar{a}_2), (\bar{a}_1, \bar{a}_2)\}$. X could be the number of defective transistors, i.e.,

  $$X((a_1, a_2)) = 0,\ X((\bar{a}_1, a_2)) = 1 = X((a_1, \bar{a}_2)),\ X(\bar{a}_1, \bar{a}_2) = 2.$$

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$$P(X = k) = \binom{15}{k} \left(\frac{1}{6}\right)^{k} \left(\frac{5}{6}\right)^{15-k},$$

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Binomial distribution

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  For example: rainfall measurements, lifetimes of components, lengths, etc. (at least in principle continuous).

Probability Mass Function - Discrete R.V.

Definition (Probability mass function)

For a **discrete** random variable X, the function $P(X = x)$ is called the **probability mass function** (pmf) of X. We have for any $A \subseteq \Omega = \{x_1, \ldots, x_n\}$,

$$P(X \in A) = \sum_{x \in A} P(X = x) = \sum_{x_i \in A} P(X = x_i) = \sum_{x_i \in A} p_i,$$

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The probability mass function (pmf) of X is

x	1	2	3	4	5	6	Σ
$p(x) = P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

Example: Maximum of two fair dice

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The probability mass function (pmf) of Y is

y	1	2	3	4	5	6	Σ
$p(y) = P(Y = y)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$	1

What is $P(Y > 4)$?

$P(Y > 4) = P(Y = 5) + P(Y = 6) = \frac{9+11}{36} = \frac{5}{9}$.

Probability Density Function - Continuous R.V.

Definition (Probability density function)

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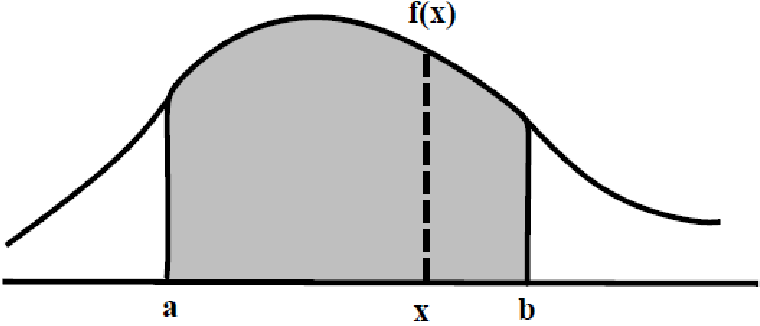

Example:

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$$\int_{x\in\Omega} f(x)\mathrm{d}x = \int_0^{350} e^{-\frac{(x-350)}{350}}\,\mathrm{d}x$$
$$= 350\left(e^{-\frac{(0-350)}{350}} - e^{-\frac{(350-350)}{350}}\right)$$
$$= 350\left(e^{1} - 1\right) \approx 601 \neq 1$$

$\longrightarrow$ f is not a probability density function.

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  $\longrightarrow$ f is a probability density function.

Summary: Probability Mass/Density Function

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pdf f

$P(a \leq X \leq b) = \int_a^b f(x)\,\mathrm{d}x$

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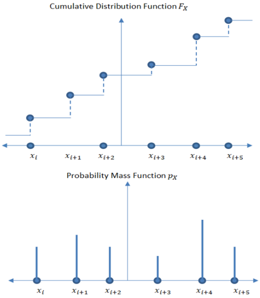

Source:
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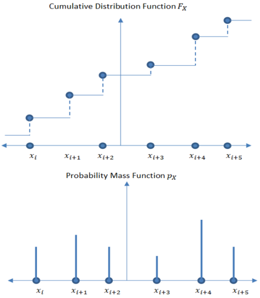
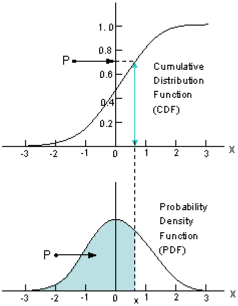


Source: http://reliability.srv.ualberta.ca/fundamental-probability-concepts-theory/random-variables



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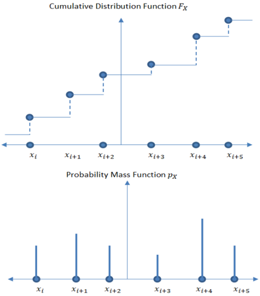
X discrete R.V.



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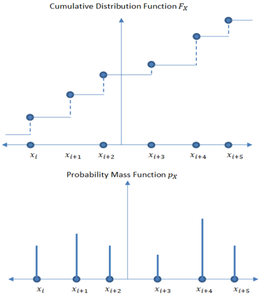

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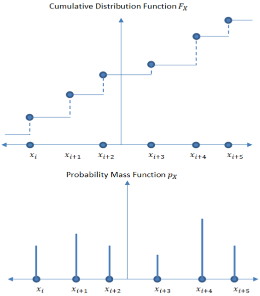

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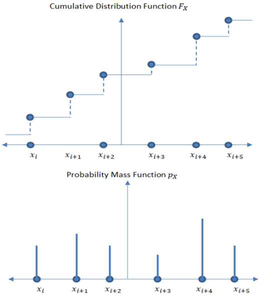

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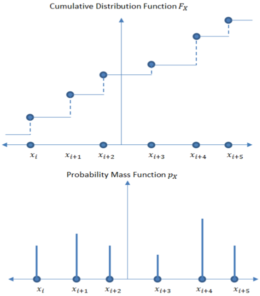

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- The height of the jump at x_i is p_i.

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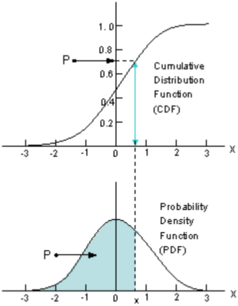

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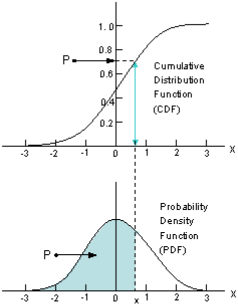

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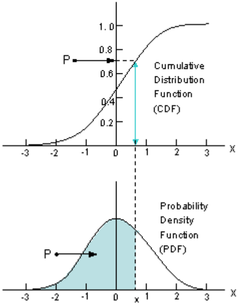

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Observation:

- cdf F is continuous.
- By definition of F
  $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,\mathrm{d}x$, so

  $f(x) = \frac{\mathrm{d}}{\mathrm{d}x} F(x).$

Example:

Draw a random number from the interval of real numbers [1, 3].
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What is the cdf F of X?

For example

$$F(2.8) = P(X \leq 2.8) = P(1 \leq X \leq 2.8) \overset{\text{geometric prob.}}{=} \frac{2.8-1}{3-1} = \frac{1.8}{2}$$

to be continued next lecture.