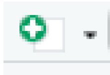
Class Example 1 – Hello World in R

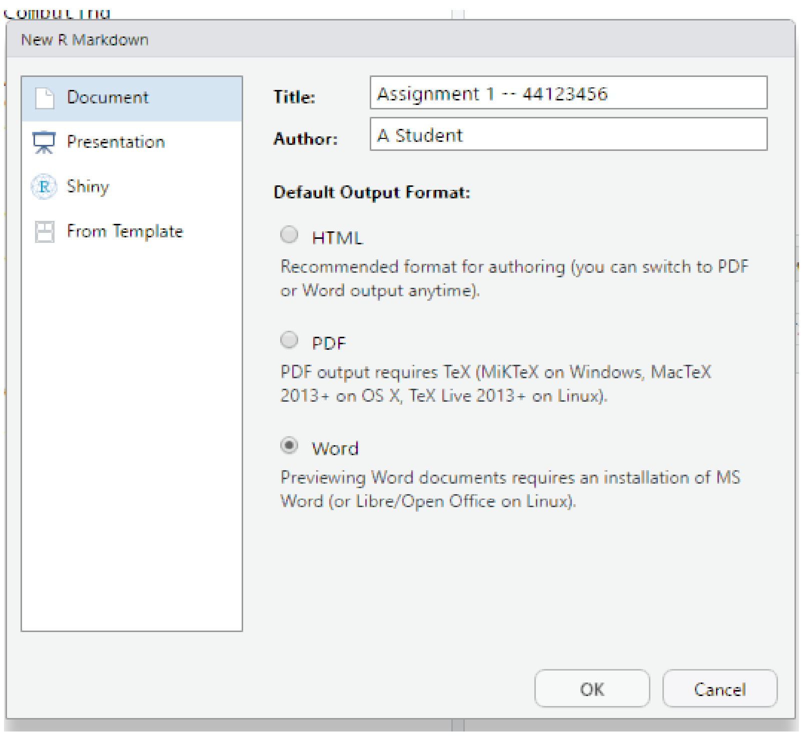
The purpose of this exercises is to gain familiarity with the R interface.

(a) Open RStudio.

(b) Start a new markdown document by clicking 

(c) In the dialog box that appears give the R Markdown document the title "Assignment 1 – <student number>" (replace student number with your own student number) and in Author write your name.

(d) Click Words as the default output format. Click okay.



(e) Click the knit button.

The R markdown document is extremely flexible, as you can included code chunks which contain executable code, text using markdown and images. You can even have the code chunks output plots in your document (as you have seen above). Markdown allows for fast formatting of text, and provides the ability to include scientific formulae using "LaTeX" syntax (you are not expected to learn LaTeXfor this course however). Your RMarkdown document demonstrates all these qualities already.

(f) Start a new R code block and solve $1 + 1$

(g) Take a photo of handwritten material with your cellular device or similar. Then upload the photo to your computer to the same directory as the RMarkdown file.

(h) Insert the photo into your RMarkdown file using `![](<filename>)`

(i) Knit your file

Class Example 2 – The Sum of Two Dice

You are rolling two independent, fair, six sided dice. Answer each of the following (i) Analytically, (ii) Using probability calculations in R, (iii) Using Monte-Carlo in R.

(a) What is the probability of the sum of the outcomes being even?

(b) What is the probability of the product of the outcomes being even?

(c) What is the set of the least likely outcomes for the sum?

Solution:

(a) (i) The table below enumerates all possible outcomes and contains the sum associated with each outcome. The even sums are shaded. There are 36 outcomes in total of which 18 are even. Since all outcomes have equal probability,

$$\text{Probability of even sum} = \frac{\text{Number of even sum outcomes}}{\text{total number of outcomes}} = \frac{18}{36} = 0.5.$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(ii) We can calculate this numerically in R as follows:

```
> sum <- 0
> for (i in 1:6) {
+         for (j in 1:6) {
+                 sum <- sum + 1 - (i+j)%%2
+         }
+ }
> prop <- sum/36
> prop
```

(iii) We can use a Monte Carlo simulation to approximate this in R as follows:

```
> values = seq(1:6)
> mean(replicate(10^6,1-(sample(values,1)+sample(values,1))%%2))
```

(b) (i) Using similar counting reasoning as in (a)(i), observe the table below to see that the probability of the product of the outcomes being even is 0.75.

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

(ii) We can calculate this numerically in R as follows:

```
> sum <- 0
> for(i in 1:6) {
+         for (j in 1:6) {
+                 sum <- sum + 1 - (i*j)%%2
+         }
+ }
> prop <- sum/36
> prop
```

(iii) We can use a Monte Carlo simulation to approximate this in R as follows:

```
> values <- seq(1:6)
> mean(replicate(10^6,1-(sample(values,1)*sample(values,1))%%2))
```

(c) We can consider outcomes of the experiment as being the sum of the dice. Then the outcomes are $\{2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12\}$. In this case the probability of each outcome is no longer equal (as is for a roll of a single die). By looking at the table of (a)(i) we see that the following probabilities hold:

$$
\begin{aligned}
P(\text{Sum is } 2) &= \frac{1}{36}, \\
P(\text{Sum is } 3) &= \frac{2}{36}, \\
P(\text{Sum is } 4) &= \frac{3}{36}, \\
P(\text{Sum is } 5) &= \frac{4}{36}, \\
P(\text{Sum is } 6) &= \frac{5}{36}, \\
P(\text{Sum is } 7) &= \frac{6}{36}, \\
P(\text{Sum is } 8) &= \frac{5}{36}, \\
P(\text{Sum is } 9) &= \frac{4}{36}, \\
P(\text{Sum is } 10) &= \frac{3}{36}, \\
P(\text{Sum is } 11) &= \frac{2}{36}, \\
P(\text{Sum is } 12) &= \frac{1}{36}.
\end{aligned}
$$

Observe that the sum of all probabilities is 1. We thus see that obtaining either 2 or 12 are the least likely events.

Class Example 3 – Cellular Phones for All

An Australian not-for-profit organization is collecting cellular devices for shipment to schools in a third world country (the devices will be used for education). In an initial sample, 1000 working devices are collected and are tested for both,

$$A \equiv \text{Surface Flaws}; \quad \text{and} \quad B \equiv \text{Defective Camera}$$

The results are as follows:

	A	$\overline{\mathbf{A}}$
B	109	364
$\overline{\mathbf{B}}$	468	59

Using the above sample, give an estimate for the following:

(a) $P(A \cap B)$ (also denoted, $P(A, B)$).

(b) $P(A \mid B)$.

(c) $P(A)$ and $P(B)$.

(d) The company plans now to collect 100,000 devices. How many devices are expected to be collected without surface flaws and a working camera?

(e) Given that a device is collected with no surface flaws, what is the chance that it has a defective camera?

Solution:

(a) Probability of Surface flaws and Defective camera: $\frac{109}{1000} = 0.109$.

(b) Probability of surface flaws, given a defective camera: $\frac{109}{(109+364)} = 0.2304$. Note that in the course, we give answers of probabilities with a precision of four decimal places.

(c) Probability of Surface flaws: $\frac{(109+468)}{1000} = 0.577$.
Probability of Defective Camera: $\frac{(109+364)}{1000} = 0.473$.

(d) $100,000 \times P(A \cap B) = 100,000 \times \frac{59}{1000} = 5,900$.

(e) $P(B \mid \overline{A}) = \frac{364}{(364+59)} = 0.8605$.